



## TSUKAMOTO FLC MODELING OF A CRISP MATHEMATICAL EEG SIGNALS MODEL AND ITS GENERALIZATION

### Abstract

In this paper we consider a system (plant) of “Hudgkin - Huxley classical mathematical model of EEG signals” as an input-output map  $y = f(x)$ . We assume that the internal structure of this system is unknown, but qualitative knowledge about the behavior is available in the form of "if - then" rules. We construct a mathematical description of the system, based on available information, so that it will represent faithfully the true system of “Tsukamoto Fuzzy Control Model”. The construction process consist of translating linguistic rules into mathematical expression using fuzzy sets and fuzzy logic using the technique of Tsukamoto fuzzy inference rules so that desired output result is achieved.

Further we generalize this model by making  $\pm 10\%$ ,  $\pm 20\%$ , etc. variations in the input sensor readings and achieve the expected output results.

The obtained Tsukamoto fuzzy controlled model is shown to be within the class of designs capable of approximating the true input- output relation to the required degree of accuracy.

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**Key words:** Mathematical model of EEG signals, inputs – output linguistic variables, Tsukamoto fuzzy rule base, weighted average formula.

### 1. INTRODUCTION

**A conventional PID Controller:** A conventional (classical) proportional-integral-derivative (PID) controller of Hodgkin-Huxley mathematical model of EEG signal is based on a rigorous mathematical model of some linear process. It reads a sensor value, applies mathematical model and produces desired output following the mathematical algorithm.

It is to be noted that the conventional mathematical EEG signal model is deceptively complex. It run up against computationally complex problems that they simply could not address without consuming prohibitive amount of computer power - if they address them at all. To overcome all such inconveniences, the need of Tsukamoto Fuzzy Controlled Model is essential.

**Tsukamoto Fuzzy logic controller (FLC):** It serves the same function as the conventional PID controller. PID manages a complex control surface by reading sensor information, executing a mathematical model and making changes to the device actuators. However the fuzzy logic controller manages this complex control surface through heuristic rather than a mathematical model.

In the "Tsukamoto fuzzy reasoning method" the consequent part of each fuzzy 'if - then' rule is represented by a fuzzy set with a monotonic membership function as shown in **Figure 1**. This method is a special case of Mamdani (both use fuzzy if - then rules whose antecedent part as a fuzzy singleton) and Takagi-Sugeno-Kang (TSK) (both use inference analogous to the weighted sum to aggregate the conclusion of multiple rules in to a final conclusion) direct fuzzy reasoning methods. Tsukamoto fuzzy model like a classical EEG signal model is based on the inputs (I/Ps) process and output (O/P) flow concepts. It requires fewer rules than classical EEG signal model and these rules (formed using linguistic variables) are closer to the knowledge which is expressed in natural language. Because the practical merits of Tsukamoto fuzzy model have been recognized over classical EEG signal model, the Tsukamoto method have been applied very effectively to provide O/P result which is as good as the O/P result of classical EEG

signal model.

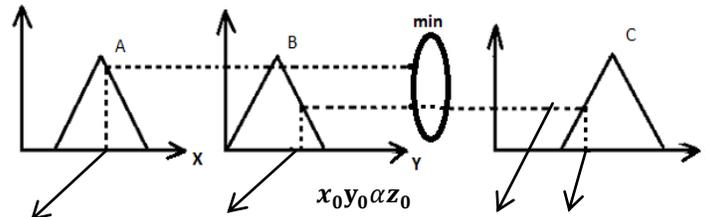


Figure 1: Tsukamoto fuzzy reasoning method representing monotonic consequent part and  $\alpha$  is the minimum matching degree between  $A(x_0)$  and  $B(y_0)$ .

**2 CLASSICAL MATHEMATICAL MODEL OF EEG SIGNALS:** This EEG signal model is based on the Hodgkin - Huxley **Nobel prize** winning model for the squid axon published in 1952<sup>[6]</sup>.

**2.1) Mechanism:** A nerve axon may be stimulated and the activated sodium ( $Na^+$ ) and potassium ( $k^+$ ) channels produced in the vicinity of the cell membrane may lead to the electrical excitation of the nerve axon. Prominently, the electrical excitation arises: (a) from the effect of membrane potential on the movement of ions, and (b) from interaction of the potential with the opening and closing of voltage activated membrane channels. The membrane potential increases when the membrane polarized with a net negative charges lining in the inner surface and equal but apposite net positive charge on the outer surface. This potential (E) may be related to the amount of electrical charge (Q), by the relation,

$$E = \frac{Q}{c_m}, \quad (1)$$

where E, electrical potential (or membrane potential or electrical force) is measured in the unit of volts; Q, electrical charge is measured in terms of coulombs/ $cm^2$ ; and  $C_m$ , is the measure of capacity of membrane in units of farad/ $cm^2$ .

In practice, in order to model the action potential (APs) the amount of charge  $Q^+$  on the inner surfaces (and  $Q^-$  on the outer surface) of the cell membrane has to be mathematically related to the stimulating current ( $I_{steam}$ ) flowing into the cell through the stimulating electrodes. The Hodgkin-Huxley model is shown in **Figure 2**.

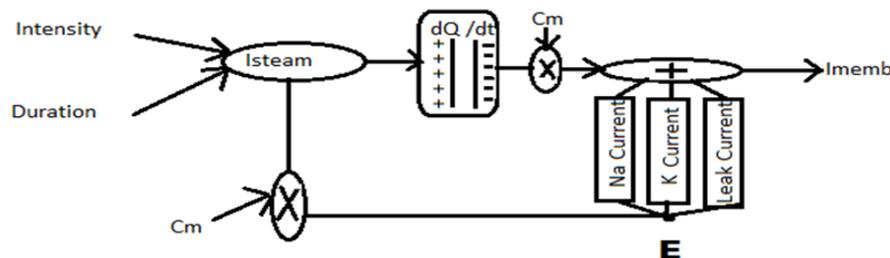


Figure 2: Hodgkin-Huxley excitation model.

In this Figure 2 membrane current ( $I_{memb}$ ) is the result of positive charges flowing out of cell. The current consists of three currents namely, sodium (Na), potassium (K) and leak currents (the leak current is due to fact that

the inner and outer Na and K ions are not exactly equal).

Hodgkin and Huxley estimated the activation and inactivation functions for the Na and K currents and derived a mathematical model to describe an action potential AP similar to that of a giant squid. The model is neuron model that usages voltage gated channels. This model describes the change in membrane potential (E) with respect to time. The overall membrane current is the sum of capacity current and ionic current as follows,

$$I_{memb} = c_m \frac{dE}{dt} + I_i \dots \dots \dots (2)$$

Where  $I_i$  is the ionic current as indicated in Figure 2. It consists of the sum of three individual components as follows,

$$I_i = I_{Na} + I_k + I_{leak} \dots \dots \dots (3)$$

where  $I_{Na}$  can be related to the maximal conductance  $\bar{g}_{Na}$ ; activation variable  $a_{Na}$ ; inactivation variable  $h_{Na}$  and driving force  $(E - E_{Na})$  through

$$I_{Na} = \bar{g}_{Na} h_{Na} a_{Na}^3 (E - E_{Na}) \dots \dots \dots (4)$$

Similarly  $I_k$  and  $I_{leak}$  can be described.

The change in the variables  $a_{Na}$ ,  $a_k$  and  $h_{Na}$  vary from 0 to 1 (time in ms) according to the following equations:

$$\frac{d}{dt}(a_{Na}) = \lambda_t [\alpha_{Na}(E)(1 - a_{Na}) - \beta_{Na}(E)a_{Na}] \dots \dots \dots (5)$$

where,  $\alpha(E)$  and  $\beta(E)$  are forward and backward rate functions respectively and  $\lambda_t$  is a temperature dependent factor.

Similarly,  $\frac{d}{dt}(h_{Na})$  and  $\frac{d}{dt}(a_k)$  can be described. The forward and backward parameters were empirically estimated by Hodgkin and Huxley as follows:

$$\alpha_{Na}(E) = \frac{3.5 + 0.1E}{1 - e^{-(3.5 + 0.1E)}}, \quad \beta_{Na}(E) = 4e^{\frac{-(E+60)}{80}}, \text{ etc.} \dots \dots \dots (6)$$

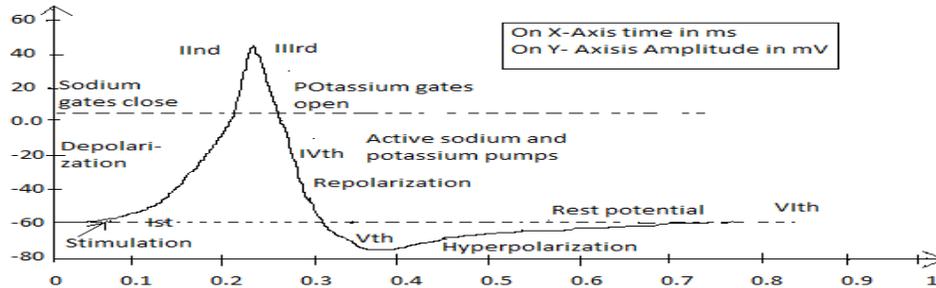
As stated in the simulator for neural network and action potential (SNNPA) literature<sup>[6]</sup>. The parameters  $\alpha(E)$  and  $\beta(E)$  have been converted from the original Hodgkin-Huxley version to a version agreeing with physiological practice taking depolarization of the membrane as positive. Resting potential has been shifted to -60mV (from original 0mV). A simulated action potential is illustrated in Figure 1. For this model, the parameters are set to be,  $c_m = 1.1 \mu F/cm^2$ ,  $\bar{g}_{Na} = 100 ms/cm^2$ ,  $\bar{g}_k = 35 ms/cm^2$ ,  $\bar{g}_l = 0.35 ms/cm^2$ ,  $E_{Na} = 60mV$ .

Using the values of  $c_m$ ,  $\bar{g}_k$ ,  $\bar{g}_l$  etc in the above related equations (1) - (6), one gets

$$I_{memb} = 80 \mu A/cm^2, \dots \dots \dots (7)$$

which is shown in Figure 3 of neuron model.

**2.2) Brief algorithm of EEG signal modeling:** The information transmitted by nerve in the central nerves system (CNS) is called an action potential (AP). APs are caused by an exchange of ions across the neuron membrane and are a temporary change in the membrane potential that transmitted along the axon. As soon as the stimulus strength goes above the threshold, an action potential appears and travels down the nerve. The membrane potential **depolarizes** (becomes more positive) producing spike. After the peak of the spike (having sodium (+) channels close and the potassium (+) open), the membrane potential **repolarizes** (becomes more negative). The potential becomes more negative than the resting potential is called **hyper polarization** and return to the normal called **resting potential** as shown in Figure 3. It is important to note that the action potential of the most nerves system last up to 5 to 10 ms.



**Figure 3: A single AP in response to a transient stimulation based on Hodgkin –Huxley model. The initiated time is  $t = 0.4\text{ms}$  and the injected current i.e.  $I_{memb} = 80\mu\text{A}/\text{cm}^2$  for duration of  $0.1\text{ms}$ .**

This model is complex due to imprecise linguistic I/P-variables and coupling of various parameters. The technique of Tsukamoto-fuzzy controllers on EEG signal modeling is more convenient under these conditions.

**3. TSUKAMOTO FUZZY CONTROLLER ON EEG SIGNAL MODEL:** The system of the classical EEG signal model consist of two fuzzy I/ Ps intensity (I) and duration ( $\tau$ ) as the stimulator for dendrites of the nerve cell and one fuzzy o/p namely membrane current ( $I_{memb}$ ) to be computed. A general scheme for controlling a desired value by the technique of Tsukamoto - FLC over the classical EEG signal model is shown by block diagram as in **Figure 3**.



Figure 3: A general scheme of Tsukamoto - FLC for controlling desired value.

To execute Tsukamoto fuzzy model the following steps are required:

- Construction of fuzzy sets and Fuzzification;
- Formation of fuzzy inference rules;
- Measurement of the adaptability and infer the conclusions;
- Aggregate the individual conclusion to obtain the overall conclusion.

Now we will see execution of these steps one by one as follows:

**(a) Construction of fuzzy sets and Fuzzification:** After identifying the relevant I/Ps and O/p variables of the classical controller, our first step in designing the FLC should be to characterize the range of values for the I/Ps and O/P variables. **Since the duration of the action potential of a nerve system in the classical controller is in the range of 5 to 10ms**, so that we have chosen the range of values for the both I/P- variables: ‘intensity’ and ‘duration’ in the time interval of 0 to 10ms in FLC. **And since final injected current in EEG signal model is,  $I_{memb} = 80\mu\text{A}/\text{cm}^2$** , accordingly we have chosen range of values for O/P- variable ‘ membrane current’ as 0 to 100  $\mu\text{A} / \text{cm}^2$  in FLC. Also we have to select meaningful linguistic states for each variable and express them by appropriate fuzzy sets. Accordingly it is assumed that the following seven linguistic states with their corresponding numerical descriptions are chosen for each of three variable: Negative Large(NL)  $\approx$  “about and below 0.13”;

Negative Medium (NM)  $\approx$  “about 0.26”; Negative Slow(NS)  $\approx$  “about 0.39”; Almost zero(AZ)  $\approx$  “about 0.52”; Positive Slow(PS)  $\approx$  “about 0.65”; Positive Medium(PM)  $\approx$  “about 0.78” and Positive Large(PL)  $\approx$  “about 0.91”.

Representing these seven linguistic states of I/P and O/P linguistic variables by triangular shape fuzzy numbers as in Figure 5 and Figure 6 respectively.

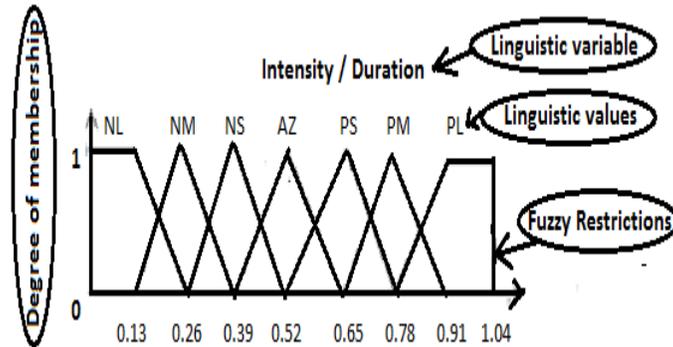


Figure 5: Fuzzy sets and decomposition for I/P variable intensity/ duration over the range [0, 1]-is the time in ms.

Next, the O/P-linguistic variable membrane current is shown in Figure 6.

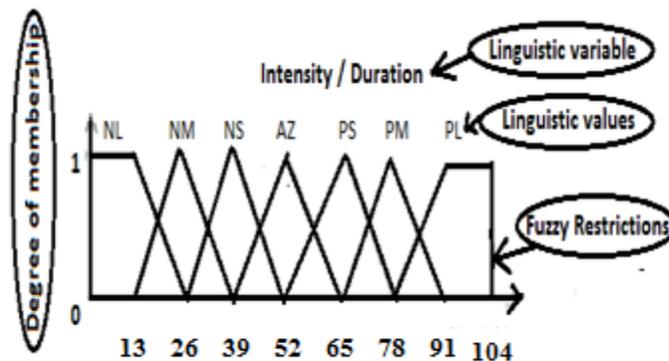


Figure 6: Fuzzy sets and decomposition for O/P variable ‘membrane current’ ( $I_{memb}$ ) over the range [0,100] is the injected current in  $\mu A/cm^2$ .

**Fuzzification of I/P-variables:-**The main purpose of the fuzzification is to interpret measurement of I/P -variables (each expressed by the fuzzy approximation of the respective real number) and to express the associated measurement uncertainties. For an illustration. A fuzzification process (function) applied to the I/P variable ‘intensity’ (I), is represented by  $f_I$ . Then the fuzzification function has the form  $f_I: [0,1] \rightarrow R$ , where R denote the set of all fuzzy numbers. Then  $f_I(x_0 = 0.40)$  is a fuzzy number chosen by  $f_I$  as a fuzzy approximation of the measurement (sensor reading) intensity (I) at  $x_0 = 0.40$ .

The computation of fuzzy membership values from Figure 5, for which  $f_I(x_0 = 0.40) \neq 0$ , is as below and shown in Figure 7.

$$NS(0.40\text{sec}) = \frac{0.40 - 0.52}{0.39 - 0.52} = \frac{0.12}{0.13} = 0.92; \quad AZ(0.40\text{sec}) = \frac{0.40 - 0.39}{0.52 - 0.39} = \frac{0.01}{0.13} = 0.08.$$

Remaining all fuzzy membership values (from Figure 4) are zero such as,  
 $NL(0.40) = NM(0.40) = PS(0.40) = PM(0.40) = PL(0.40) = 0$ .

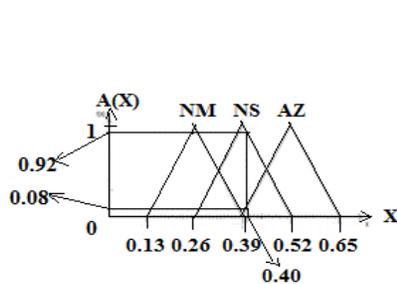
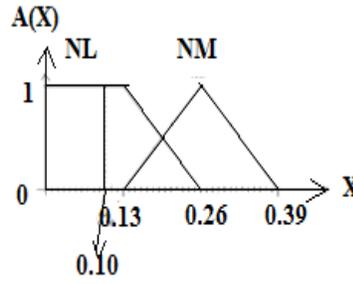


Figure (7)



Figure(8)

Figures (7 and 8): Fuzzification of I/P variables intensity for  $x_0 = 0.40$  and duration  $y_0 = 0.10$  is shown in Figure 7 and Figure 8 respectively.

The computation of fuzzy membership values from Figure 6 for which  $f_\tau(y_0 = 0.10) \neq 0$ , is as below and their Pictorial representation is as shown in Figure 8.

The membership values for fuzzy sets NL are computed as,  
 $NL(0.10) = 1$ .

All other remaining membership values from Figure 6 are zero. Such as  $NS(0.10) = AZ(0.10) = PL(0.10) = PM(0.10) = PS(0.10) = NM(0.10) = 0$ . This shows that only one rule fires, namely  $NL(0.10) = 1$ .

**(b)Formation of fuzzy inference rules:** - The knowledge pertains to the given control problem is formulated in terms of a set of fuzzy inference rules. To elicit Tsukamoto fuzzy inference rules, for the I/P-variables intensity (I), duration ( $\tau$ ) and O/P -variable membrane current ( $I_{memb}$ ) in our control problem, the rule base have the canonical form,

$$\text{“If } I = A_i \text{ and } \tau = B_i \text{ then } I_{memb} = C_i\text{”}, I = 1, 2, \dots, n \quad \dots \dots (8)$$

where  $A_i, B_i$  and  $C_i$  are fuzzy numbers chosen from the set of fuzzy numbers (on the domains X, Y & Z- axes respectively) that represent the linguistic states NL, NM, NS, AZ, PM, PS and PL and  $\mu_{C_i}(z)$  is a monotonic function. Since each I/P- variable has, seven linguistic states, the total number of possible non- conflicting fuzzy inference rules are  $7^2 = 49$ .

In practice, instead of these 49 rules, a small subset of all possible fuzzy inference rules is often sufficient to obtain acceptable performance of the fuzzy controllers.

An appropriate subset of fuzzy rules derived intuitively by common sense reasoning is as follows:

- Rule (1): If I is AZ and  $\tau$  is NL then  $I_{memb}$  is PL
- Rule (2): If I is NS and  $\tau$  is NL then  $I_{memb}$  is PM
- Rule (3): If I is NM and  $\tau$  is NL then  $I_{memb}$  is NS
- Rule (4): If I is NM and  $\tau$  is AZ then  $I_{memb}$  is AZ
- Rule (5): If I is NS and  $\tau$  is PS then  $I_{memb}$  is PL
- Rule (6): If I is PS and  $\tau$  is NS then  $I_{memb}$  is PS
- Rule (7): If I is PL and  $\tau$  is AZ then  $I_{memb}$  is PL
- Rule (8): If I is AZ and  $\tau$  is NS then  $I_{memb}$  is PS

Rule (9): If I is AZ and  $\tau$  is NM then  $I_{memb}$  is PM

**(c) Measurement of the adaptability and infer conclusions:** - Measurements of I/P-variables of fuzzy controller must be properly combined with the relevant fuzzy information rules to make inference regarding the O/P- variables. This is the purpose of the inference engine. This process of finding inferred crisp O/P by inference is called rule strength computation or adaptability the rule or firing strength. We note that in the Tsukamoto fuzzy rules given by (8), the consequence part of each rule is represented by fuzzy set  $C_i$  with monotonic membership function  $\mu_{C_i}(w)$  and that  $\alpha_i$  is the matching degree of the  $i$ th rule. For the singleton input values (sensor readings) of the linguistic variables intensity ( $I = x_0$ ) and duration ( $\tau = y_0$ ) the matching degree  $\alpha_i$  is obtained by

$$\alpha_i = \mu_{A_i}(x_0) \wedge \mu_{B_i}(y_0), i = 1, 2, \dots, n$$

Where “ $\wedge$ ” denote the minimum operation.

The overall inferred O/P result is taken as the weighted average of each rule’s output is given by

$$w_i = \mu_{C_i}^{-1}(\alpha_i), i = 1, 2, \dots, n \quad (9)$$

The final result is derived from the weighted average formula which is expressed as,

$$w_0 = \frac{\sum_{i=1}^n \alpha_i w_i}{\sum_{i=1}^n \alpha_i}, \text{ where } n \text{ is a finite positive integer.}$$

Since each rule infers a crisp result, the Tsukamoto fuzzy model aggregates each rule’s O/P by the weighted average method. Therefore, this method avoids the time consuming process of defuzzification.

Following the above mathematical steps of the Tsukamoto fuzzy rule base for the computation final o/p result we proceed as;

Utilizing fuzzy membership values from Figure 7 and Figure 8 and appropriate subset fuzzy rules that fired only (1 and 2).

We write these rules for sensor reading  $(x_0, y_0) = (0.40, 0.10)$  of the I/P variables intensity and duration respectively.

Rule (1): If  $x [= I=0.40]$  is  $A_1[AZ=0.08]$  and  $y [= \tau = 0.10]$  is  $B_1[=NL=1]$  then  $z [ I_{memb} ]$  is  $C_1 [=PL]$ .

Rule (2): If  $x [= I=0.40]$  is  $A_2[NS=0.92]$  and  $y [= \tau = 0.10]$  is  $B_2[=NL=1]$  then  $z [ I_{memb} ]$  is  $C_2[=PM]$ .

The computation for measure of adaptability of each rule is as follows:

Adaptability Rule (1):  $\alpha_1 = \mu_{A_1}(x_0 = 0.40) \wedge \mu_{B_1}(y_0 = 0.10) = \min(0.08, 1) = 0.08$

Adaptability Rule (2):  $\alpha_2 = \mu_{A_2}(x_0 = 0.40) \wedge \mu_{B_2}(y_0 = 0.10) = \min(0.92, 1) = 0.92$

Where “ $\wedge$ ” represents minimum -operation.

We can check very easily adaptability of remaining six rules are zero:  $\min(0, 0) = \min(0, 0) = \min(0.920, 0) = \min(0, 0) = \min(0, 0) = \min(0.0799, 0) = 0$ .

The calculations in the conclusion rules 1 and 2 corresponds with cutting the fuzzy sets in the consequence part by height of the adaptability of the premise part are shown in Figure 9.

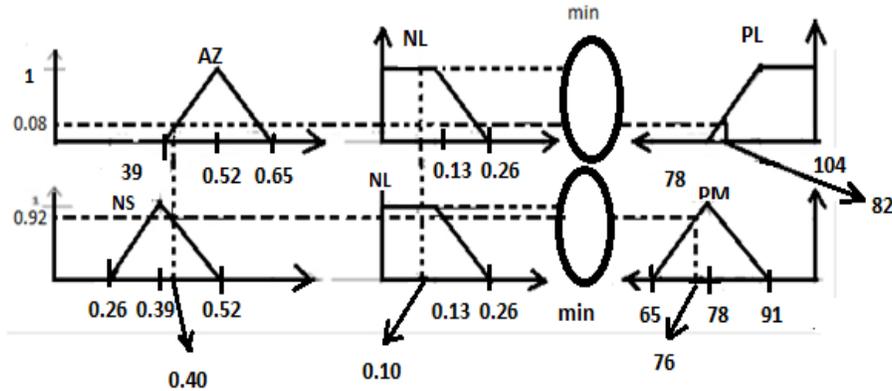


Figure 9: Graphical representation of Tsukamoto method by height adaptability of premise part.

(d) **Aggregate the individual conclusion to obtain the overall conclusion:** -Final O/P result is derived using the weighted average formula as follows, when there are two “If-Then” rules are in action,

$$w_0 = \frac{\alpha_1 w_1 + \alpha_2 w_2}{\alpha_1 + \alpha_2},$$

Now using the values  $\alpha_1, \alpha_2, w_1$  and  $w_2$  in the above equation we get,

$$w_0 = \frac{0.08 * 82 + 0.92 * 76}{0.08 + 0.92} = 76.48.$$

Thus for the singleton I/Ps  $(x_0, y_0) = (0.40, 0.10)$  of the linguistic variables intensity (I) and duration ( $\tau$ ) respectively by Tsukamoto fuzzy control we get desired O/P result i.e. membrane current ( $I_{memb}$ ) is,

$$I_{memb} = 76.48 \mu A/cm^2.$$

**4. GENERALIZATION OF TSUKAMOTO - FUZZY LOGIC CONTROL EEG SIGNAL MODEL:** In order to examine the sensitivity and validity of Tsukamoto - fuzzy logic controlled EEG signal model, we design this for distinct I/P values (sensor readings) of linguistic variables intensity (I) and duration ( $\tau$ ) and study responses of the O/P results “membrane current” of the respective model. This is to be carried out by the following three steps:

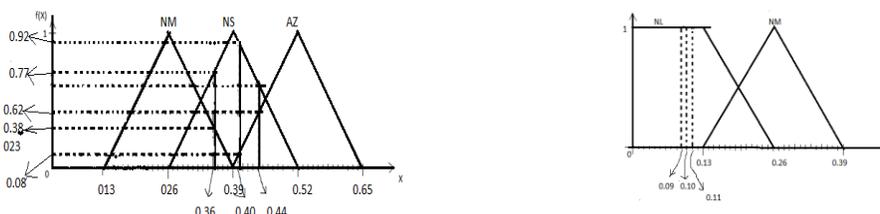
**Step (a<sub>1</sub>):** Fuzzification of I/P linguistic variables for distinct sensor readings.

**Step (b<sub>1</sub>):** Measurement of adaptability and infer the conclusion.

**Step (c<sub>1</sub>):** Aggregation the individual conclusion to obtain the overall conclusion.

**Step (a<sub>1</sub>):** Fuzzification of I/P linguistic variables for distinct sensor readings.

The computation of fuzzy membership values from Figure 5 for which  $f_i(x_0 = 0.40) \neq 0$ , is already calculated in **step (a)** and its Pictorial representation is also shown in Figure 9.



**Figure 10 Figure 11**

Figures (10 and 11): *The fuzzification of I/P- variables intensity (at  $x_0 = 0.40$  and its  $\pm 10\%$  variations) and duration (at  $y_0 = 0.10$  and at its  $\pm 10\%$  variations) is shown in Figure 10 and Figure 11 respectively.*

In order to examine the sensitivity responses of O/P results of fuzzy controller, we calculate the membership values for the respective fuzzy sets by varying  $\pm 10\%$  of the above sensor reading  $x_0 = 0.40$  as follows.

First maximizing 10% of 0.40 we get 0.44. The determination of the membership values for NS and AZ is as below and is shown in Figure 10.

$$NS(0.44) = \frac{0.44-0.52}{0.39-0.52} = \frac{0.08}{0.13} = 0.620; AZ(0.44) = \frac{0.44-0.39}{0.52-0.39} = \frac{0.05}{0.13} = 0.380.$$

Remaining all fuzzy membership values are zero such as,  $NL(0.44) = NM(0.44) = PS(0.44) = PM(0.44) = PL(0.44) = 0$ .

Next by minimizing 10% of 0.40 we get 0.36. The determination of the membership values for NS and NM is as below and is shown in Figure 10.

$$NS(0.36) = \frac{0.36-0.26}{0.39-0.26} = \frac{0.10}{0.13} = 0.770; NM(0.36) = \frac{0.36-0.39}{0.26-0.39} = \frac{0.03}{0.13} = 0.230.$$

Remaining all fuzzy membership values are zero such as,  $NL(0.36) = AZ(0.36) = PS(0.36) = PM(0.36) = PL(0.36) = 0$ .

Proceeding similar to above. The computation of fuzzy membership values for  $f_{\tau}(y_0 = 0.10)$  is carried out using only that part of Figure 6 for which  $f_{\tau}(y_0 = 0.10) \neq 0$ , as below and is shown in Figure 11.

$$NL(0.10) = 1.$$

Remaining all memberships values from Figure 6 are zero such as,  $NS(0.10) = AZ(0.10) = PL(0.10) = PM(0.10) = PS(0.10) = NM(0.10) = 0$ . This shows that only one rule fires, namely  $NL(0.10) = 1$ .

In order to examine the sensitivity and validity of O/P results of fuzzy controller, we calculate the membership values for the respective fuzzy sets by varying  $\pm 10\%$  of the above sensor reading  $y_0 = 0.10$  as follows.

First by maximizing 10% of 0.10 we get 0.11. The determination of the membership values for the fuzzy set NL is as below and is shown in Figure 11

$$NL(0.11) = 1.$$

Remaining all memberships values from Figure 6 are zero such as,  $NS(0.11) = AZ(0.11) = PL(0.11) = PM(0.11) = PS(0.11) = NM(0.11) = 0$ . This shows that only one rule fires, namely  $NL(0.11) = 1$ .

Secondly by minimizing 10% of 0.10 we get 0.09. The determination of the membership values for the fuzzy set NL is as below and is shown in Figure 11.

$$NL(0.09) = 1.$$

Remaining all memberships values from Figure 6 are zero such as NS (0.09) = AZ (0.09) = PL (0.09) = PM (0.09) = PS (0.09) = NM (0.09) = 0. This shows that only one rule fires, namely NL (0.09) = 1.

**(c) Measurement of the adaptability and infer conclusion:** -For measurements of the adability of the premise rule for  $\pm 10\%$  variations of the values of the I/p linguistic variables and then to infer the conclusion from these adaptability we proceed as follows:

Utilizing fuzzy membership values from Figure 10 and Figure 11 and appropriate subset fuzzy rules that fired (1 and 2) we have,

We write these rules for 10% maximization of sensor reading  $(x_0, y_0) = (0.40, 0.10)$  of the I/P variables intensity and duration respectively. So that we get,

Rule (1): If x [=I=0.44] is  $A_1$ [AZ=0.38] and y [=τ=0.11] is  $B_1$ [=NL=1] then z [ $I_{memb}$ ] is  $C_1$  [=PL].

Rule (2): If x [=I=0.44] is  $A_2$ [NS=0.62] and y [=τ=0.11] is  $B_2$ [=NL=1] then z [ $I_{memb}$ ] is  $C_2$  [=PM].

The computation for measure of adaptability of each rule is as follows:

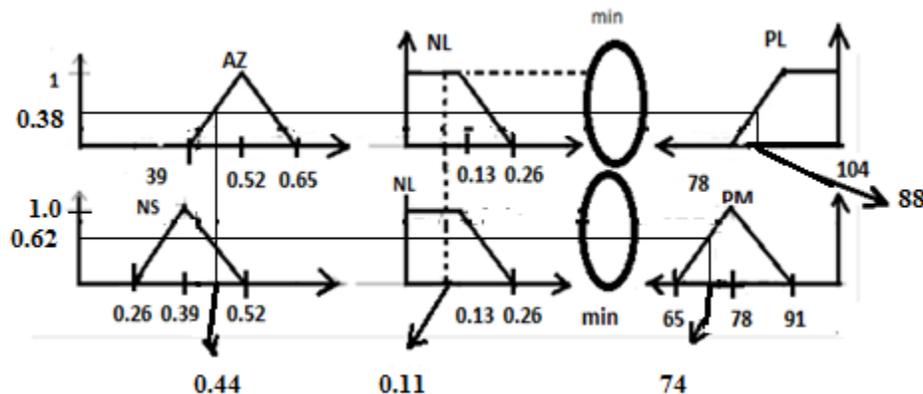
Adaptability Rule (1):  $\alpha_1 = \mu_{A_1}(x_0 = 0.44) \wedge \mu_{B_1}(y_0 = 0.11) = \min(0.38, 1) = 0.38$ ;

Adaptability Rule (2):  $\alpha_2 = \mu_{A_2}(x_0 = 0.44) \wedge \mu_{B_2}(y_0 = 0.11) = \min(0.62, 1) = 0.62$

Where “ $\wedge$ ” represents minimum -operation.

We can check very easily adaptability of remaining six rules are zero:  $\min(0, 0) = \min(0, 0) = \min(0.380, 0) = \min(0, 0) = \min(0, 0) = \min(0.62, 0) = 0$ .

The calculations in the conclusion rules 1 and 2 corresponds with cutting the fuzzy sets in the consequence part by height of the adaptability of the premise part are shown in Figure 12.



**Figure 12: Graphical representation of Tsukamoto method.**

**Aggregate the individual conclusion to obtain the overall conclusion:** - Final O/P result is derived using the weighted average formula as follows, when there are two “If-Then” rules are in action,

$$w_0 = \frac{\alpha_1 w_1 + \alpha_2 w_2}{\alpha_1 + \alpha_2},$$

Now using the values  $\alpha_1, \alpha_2, w_1$  and  $w_2$  in the above equation we get,

$$w_0 = \frac{0.38 * 88 + 0.62 * 74}{0.38 + 0.62}$$

$$= 79.32.$$

Similar to above Utilizing fuzzy membership values from figure 7 and figure 8 and appropriate subset fuzzy rules that fired only (1,2 and 3) we have,

We write these rules for 10% minimization of sensor reading  $(x_0, y_0) = (0.40, 0.10)$  of the I/P variables intensity and duration respectively. So that we get,

Rule (1): If  $x [= I=0.36]$  is  $A_1[NS=0.77]$  and  $y [=τ=0.09]$  is  $B_1[=NL=1]$  then  $z [ I_{memb} ]$  is  $C_1 [=PM]$ ;

Rule (2): If  $x [= I=0.36]$  is  $A_2[NM=0.23]$  and  $y [=τ=0.09]$  is  $B_2[=NL=1]$  then  $z [ I_{memb} ]$  is  $C_2[=PS]$ ;

The computation for measure of adaptability of each rule is as follows:

Adaptability rule-1:  $\alpha_1 = \mu_{A_1}(x_0 = 0.36) \wedge \mu_{B_1}(y_0 = 0.09) = \min(0.23, 1) = 0.23$

Adaptability rule-2:  $\alpha_2 = \mu_{A_2}(x_0 = 0.36) \wedge \mu_{B_2}(y_0 = 0.09) = \min(0.77, 1) = 0.77$

Where “ $\wedge$ ” represents minimum -operation.

We can check very easily adaptability of remaining six rules are zero:  $\min(0, 0) = \min(0, 0) = \min(0.380, 0) = \min(0, 0) = \min(0, 0) = \min(0.62, 0) = 0$ .

The calculations in the conclusion rules 1 and 2 corresponds with cutting the fuzzy sets in the consequence part by height of the adaptability of the premise part are shown in Figure 13.

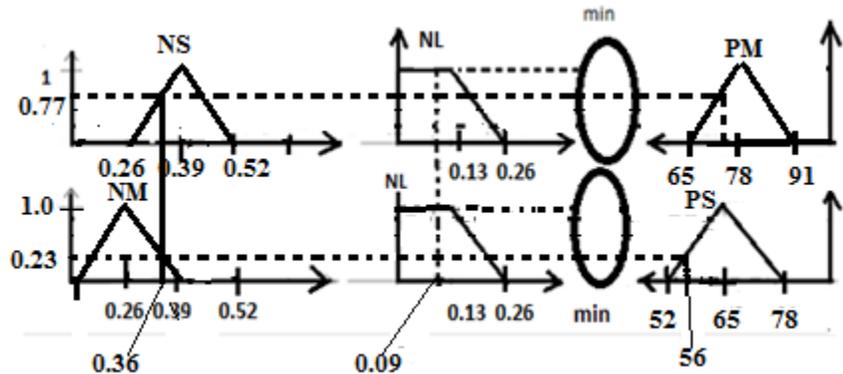


Figure 13 . Graphical representation of Tsukamoto method.

**Aggregate the individual conclusion to obtain the overall conclusion:** - Final O/P result is derived using the weighted average formula as follows, when there are two “If-Then” rules are in action,

$$w_0 = \frac{\alpha_1 w_1 + \alpha_2 w_2}{\alpha_1 + \alpha_2},$$

Now using the values  $\alpha_1, \alpha_2, w_1$  and  $w_2$  in the above equation we get,

$$w_0 = \frac{0.77 * 76 + 0.23 * 56}{77 + 0.23}$$

$$= 71.40.$$

The comparative study of O/P results of Hodgkin- Huxley classical EEG signal model and our designed Tsukamoto FLC models is given in the following table.

Models	Input Sensor Readings	Output Results
Classical EEG Signals Model	$(x_0, y_0) = (0.40, 0.10)$	80.00
FLC-EEG Signals Model	$(x_0, y_0) = (0.40, 0.10)$	76.64
FLC-EEG Signals Model	$(x_0, y_0) = (0.44, 0.11)$	79.32
FLC-EEG Signals Model	$(x_0, y_0) = (0.36, 0.09)$	71.40

### 5. CONCLUSION:

If we go through the stepwise discussions of Tsukamoto fuzzy control model comparing with classical EEG signal model. It is observed that the numbers of rules are greatly reduced and computational complexities are highly mitigated. The general modification and tuning of control rules are very easily carried out. The working skeleton and final O/P result of the model signify that the Tsukamoto – fuzzy control model have better performance in comparison to the classical mathematical model of EEG signals. The comparative result implicates that the model of the Tsukamoto -fuzzy control obtained from the classical mathematical model of EEG signal are catering the actual dynamics of the system.

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